Discussion 9A Recap

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1 Discrete Probability Distributions

A **random variable** X on a sample space Ω assigns every sample ω a real number $X(\omega)$. We can augment probability spaces with random variables now!

A **distribution** of a discrete random variable X are its values and their associated probabilities, i.e. $\{(a, \mathbb{P}(X = a)) | a \in \mathcal{A}\}$ where $\mathcal{A}$ is the set of values X takes on.

Distributions are useful because they model situations that happen all the time.

Important distributions (for now)

- Bernoulli: coin flip or a success
- Binomial: n coin flips or trials
- Geometric: number of trials until a success

Each of them has some **parameters**, which just specify values that you can adjust for your application (like probability of a success, number of total flips, etc.). Be familiar with their intuitive explanations as they can save you lots of time when reasoning about the behavior of new distributions built from these ones.

Bernoulli distributions are random variables where $X = 1$ if a trial succeeds or a coin comes up heads.

- Parameter: probability p of success.
- Probability Mass Function (PMF): $\mathbb{P}(X = 1) = p$, $\mathbb{P}(X = 0) = 1 - p$.
- Denoted as $X \sim \text{Ber}(p)$.

Binomial distributions are random variables where $X =$ the number of successful trials in n trials. They're esentially just n independent Bernoulli trials.

- Parameter: n number of trials, each with probability p of success.
- PMF: $\mathbb{P}(X = i) = \binom{n}{i} p^i (1-p)^{n-i}$, $0 \leq i \leq n$.
- Denoted $X \sim \text{Bin}(n, p)$.

Geometric distributions are random variables where $X =$ the number of trials until a successful trial.

- Parameter: probability p of success.
- PMF: $\mathbb{P}(X = i) = (1-p)^{i-1} p$, $i \geq 1$
- Cumulative Mass Function (CMF): $\mathbb{P}(X \leq i) = 1 - (1-p)^i$.
- Denoted $X \sim \text{Geo}(p)$.
- Only discrete distribution which is **memoryless**, i.e. $\mathbb{P}(X > m + n | X > n) = \mathbb{P}(X > m)$. In other words, how long you've waited doesn't affect the probability of your event.