Discussion 8A Recap

Tyler Zhu

April 4, 2020

1 Conditional Probability

Most of this should make intuitive sense. Throughout this list, let A, B be events in some probability space $(\Omega, \mathbb{P})$.

- **Product Rule** (Conditional Probability Rule): $\mathbb{P}(A|B) = \mathbb{P}(A \cap B)/\mathbb{P}(B)$.
  - Numerator is the event that A and B both happen, denominator is just B.
- **Disjoint** or mutually exclusive events: $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$.
  - In Venn diagram (or Inclusion-Exclusion Principle), means the probability of their intersection is empty.
- **Bayes' Rule**: Let's us compute $\mathbb{P}(A|B)$ if we know $\mathbb{P}(B|A)$: $\mathbb{P}(A|B) = \mathbb{P}(B|A)\mathbb{P}(A)/\mathbb{P}(B)$
  - Might calculate P(B) using total probability.
  - Usually events are conditioned on in a sequential order, so this let's us compute weird ones in a natural manner.
  - Direct application of product rule.
- **Total Probability**: Partition event A into cases and condition on them ("summing out").
  - $\mathbb{P}(A) = \mathbb{P}(A|B_1)\mathbb{P}(B_1) + \ldots + \mathbb{P}(A|B_n)\mathbb{P}(B_n)$ where the B_i are partitions of the sample space.
  - Probability that A happens must be the same conditioned on all ways it can happen.
- **Independence**: $\mathbb{P}(A|B) = P(A)$, i.e. event B has no effect on event A
  - Rearranging gives the usual definition $\mathbb{P}(A \cap B) = P(A) \cdot P(B)$, but I think this one is more natural.
  - **Mutual independence**: $\mathbb{P}(\cap_i A_i) = \prod_i A_i$
  - Pairwise independence does not imply mutual independence
- **Union Bound**: probability of the union of events is upper bounded by the sum of their probabilities
  - $\mathbb{P}(\cup_i A_i) \leq \sum_i \mathbb{P}(A_i)$.
  - Reason: worst case all events are mutually exclusive, so equality actually holds
  - Throws a lot of information away, but (surprisingly) in general is good enough.

2 Symmetry

Sometimes one can leverage symmetry (or exchangeability) in the problem to make it easier to solve. This usually comes in the form of treating the probability that something happens about the third coin in a sequence as if it was about the first coin (since, by symmetry, they should be the same). Has nontrivial applications.