Discussion 11A Recap

Tyler Zhu

April 22, 2020

1 Tail Bounds

We have these two important tail bounds.

Theorem 1 (Markov's Inequality)**.** Let X be any nonnegative r.v. with finite mean. Then for any $a > 0$,

$$\mathbb{P}[X \geq a] \leq \frac{\mathbb{E}[X]}{a}.$$

Theorem 2 (Chebyshev's Inequality)**.** Let X be a r.v. with finite mean and finite non-zero variance. Then for any constant $k > 0$,

$$\mathbb{P}[|X - \mathbb{E}[X]| \geq k] \leq \frac{\mathrm{Var}(X)}{k^2}.$$

We call them tail bounds because they bound the probability of the tails occuring (insightful I know). Importantly, Markov's is a one-sided tail bound, while Chebyshev's is two tailed.

2 Tips

- If you find yourself with a random variable with a maximum or minimum value, i.e. $X \geq -2$ or $Y \leq 10$, you should look to use Markov's on a transformed r.v. which is nonnegative, i.e. $X' = X + 2 \geq 0$ and $Y' = 10 - Y \geq 0$.
- Even though Chebyshev's Inequality is two-tailed, if the distribution of X is symmetric, we can make it one-tailed by dividing by 2. But only if it's symmetric (see homework problem for an extension).
- When using Chebyshev's to compute one-sided bounds like $X \geq 10$, just subtract both sides by $\mathbb{E}[X]$ to find out what k should be.
- Get used to computing confidence intervals with Chebyshev's Inequality.