

# Discussion 0A Recap

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## 1 Sets

Here's some basic notation you should be familiar with. For this section, let  $A = \{1, 2\}$  and  $B = \{1, 2, 3, 4\}$ .

- Cardinality, i.e. size of a set.  $|A| = 2$ .
- Subsets, i.e. when all members of one set belong to another.  $A \subseteq B$ .
- Intersection, i.e. everything in common.  $A \cap B = \{1, 2\}$ .
- Union, i.e. everything that appears.  $A \cup B = \{1, 2, 3, 4\}$ .
- Set difference, i.e. everything in one but not the other.  $B \setminus A = \{3, 4\}$ .
- Cartesian product, i.e. all possible pairs (or tuples) of elements:  
 $A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), \dots\}$ .
- Power set, i.e. the set of all subsets.  $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ . Note that  $|\mathcal{P}(A)| = 2^{|A|}$ .

Some important sets that you'll come across this semester:

- The naturals (which includes 0):  $\mathbb{N} = \{0, 1, 2, \dots\}$
- The integers:  $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
- The rationals:  $\mathbb{Q} = \{\frac{a}{b} | a, b \in \mathbb{Z}, b \neq 0\}$
- The reals:  $\mathbb{R}$
- The complexes:  $\mathbb{C}$

As we saw in discussion, if you want to show that two sets are equal, i.e.  $A = B$ , you need to show  $A \subseteq B$  and  $B \subseteq A$ . This is the same as showing  $x = y$  by proving  $x \leq y$  and  $y \leq x$ .

## 2 Propositional Logic

We work with logical statements, or propositions. Operators operate on propositions. The common ones are

- and,  $P \vee Q$ : True only if both propositions are True.
- or,  $P \wedge Q$ : False only if both propositions are False (unlike the English or, which is commonly either or).
- not,  $\neg P$ : Negates the truth value, i.e. True becomes False and vice versa.
- implies,  $P \implies Q \equiv \neg P \wedge Q$ : False only if  $P$  is True and  $Q$  is False.

Finally, we have two quantifiers: *for all*, which is  $(\forall x \in \mathbb{R})$ , and *there exists*, which is  $(\exists x \in \mathbb{Z})$ .

### 3 Tips

- Use DeMorgan's Laws to move negations past  $\wedge, \vee$ . Negate the clauses and swap between the two.

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

- To simplify expressions, distribute operators by following the “distributive law,” e.x.

$$(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R).$$

The operator linking  $P$  and  $Q$  remains as the operator linking the two resulting clauses.

- Operators cannot be arbitrarily switched, i.e.  $\forall x \exists y P(x, y) \not\equiv \exists y \forall x P(x, y)$ . Take  $P(x, y) = x < y$  as a counterexample.