Discussion 2A Recap

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1 Terminology

- An **instance** of a stable matching is a set of preference lists of jobs and candidates.
- A **matching** for a stable matching is a set of candidate-job couples (C_i, J_i).
- A **rogue couple** is a pair (C, J) that prefer each other over their current partners.
- A **stable** matching is a matching without any rogue couples.
- A **(job/candidate) optimal** matching is one where the (jobs/candidates) receive their highest preferences of *all* stable matchings.

The traditional propose-and-reject algorithm (PAR) has jobs propose to candidates. It provides us with a job-optimal, candidate-pessimal, stable matching.

2 Important Concepts

Recall that the propose-and-reject algorithm performs three stages every day until termination:

(a) **Morning**: Every job proposes to the best candidate who has yet to reject the job yet.

(b) **Afternoon**: Each candidate rejects all jobs proposed to her except for her favorite job, which she keeps on a string.

(c) **Evening**: Each job crosses off the candidate that rejected them, if any.

- *Candidate Improvement Lemma*: The job a candidate has on her string can only get better.
- In general, consider proofs by induction or contradiction with stable matching problems. In most contradiction proofs, use the Well-Ordering Principle ("The first day that...") and construct a rogue couple.
- One very common technique is to create preference lists where the jobs have distinct first choices so that it's easy to reason about what happens.
- A common counterexample is the 2×2 case where jobs and candidates have different preference lists.
- Counting the number of rejections occuring can help in some contradiction proofs.