Discussion 1A Recap

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1 Proofs

You should know these main proof types:

- Direct Proof: show $P \implies Q$ where P is a truth and Q is our claim.
- Contrapositive: for a statement $P \implies Q$, prove $\neg Q \implies \neg P$.
- Contradiction: to prove a claim P, assume for the sake of contradiction that $\neg P$ is true. Show this implies $R \wedge \neg R$ (for some R), contradiction. Hence P is true.
- By cases: To show P is true in general, we prove P in separate cases, which in combination cover all possible cases (i.e. cases are a partition).

If you're thinking of skipping step B in a logical sequence $A \to B \to C$ of a proof, you should ask yourself if a reader would have to think hard to deduce $A \to C$ without including B.

Proofs should also be written in grammatical English, and your proofs need not be all symbols.

2 Contrapositive vs. Contradiction

There's a subtle difference between a proof by contrapositive and a proof by contradiction that's hard to see at first. We can illustrate this with an example.

Example 2.1. Suppose you have a rectangular array of pebbles, where each pebble is either red or blue, with the following property: for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones. Prove that there must exist an all-red column.

Proof by Contrapositive. We will instead prove the contrapositive, which is:

If there is no all-red column, then there is some way of choosing one pebble from each column so that no red pebble exists among the chosen ones.

But if there is no all-red column, then there is a blue pebble in every column. Picking every such blue pebble gives us a collection with no red pebbles. □

Proof by Contradiction. Assume for the sake of contradiction that there was no all-red column. Then there is a blue pebble in every column, so picking every such blue pebble gives us a collection with no red pebbles, contradiction to the property of our array. Hence, there must be an all-red column. □

In this case, the contrapositive and the contradiction were virtually identical because the fact R used to derive the contradiction was our base assumption. In other words, the contrapositive proved the statement $P \implies Q$ (where Q is our claim) by showing $\neg Q \implies \neg P$. Contradiction proved our claim Q by assuming $\neg Q$ and logically arriving at $\neg P$, which is a contradiction with our base assumption P.

If we had used a different clause R to draw our contradiction, then the proofs would have been different.