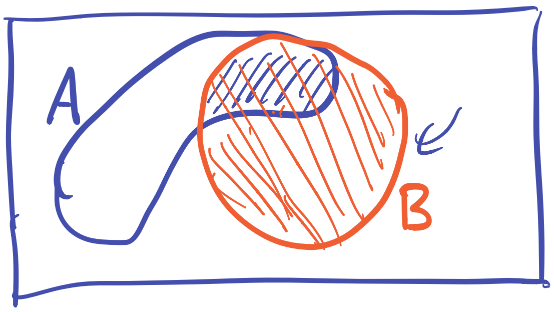
Dis 09B: Conditional Probability

Review

Conditional Probability: $\mathbb{P}(A \mid B)$, read "prob. of A given B"



there's alot of related concepts in conditional probability.

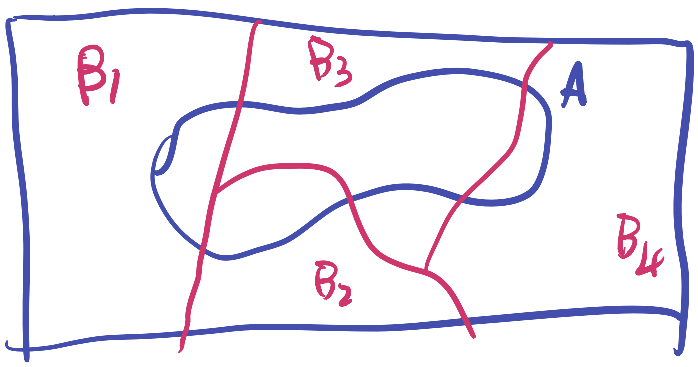
Product Rule: $\mathbb{P}(A \mid B) = \mathbb{P}(A \cap B) / \mathbb{P}(B)$ (self-evident)

↳ sometimes seen as $\mathbb{P}(A \mid B) \cdot \mathbb{P}(B) = \mathbb{P}(A \cap B)$.

Bayes' Rule: $$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)}.$$

Law of Total Probability: If $B_1, B_2, \ldots, B_n$ partition the space,

$$\mathbb{P}(A) = \mathbb{P}(A \mid B_1)\,\mathbb{P}(B_1) + \mathbb{P}(A \mid B_2)\,\mathbb{P}(B_2) + \cdots + \mathbb{P}(A \mid B_n)\,\mathbb{P}(B_n)$$



CS 70 Discrete Mathematics and Probability Theory
Fall 2020 DIS 9B

1 Box of Marbles

You are given two boxes: one of them containing 900 red marbles and 100 blue marbles, the other one contains 500 red marbles and 500 blue marbles.

(a) If we pick one of the boxes randomly, and pick a marble what is the probability that it is blue?

A: marble is blue, B_1: box 1 is picked, B_2: box 2 is picked.

$$\mathbb{P}(A) = \mathbb{P}(A \mid B_1)\,\mathbb{P}(B_1) + \mathbb{P}(A \mid B_2)\,\mathbb{P}(B_2) = \frac{1}{10}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \boxed{\frac{3}{10}}.$$

(b) If we see that the marble is blue, what is the probability that it is chosen from box 1?

$$\mathbb{P}(B_1 \mid A) = \frac{\mathbb{P}(A \mid B_1)\cdot\mathbb{P}(B_1)}{\mathbb{P}(A)} = \frac{\frac{1}{10}\cdot\frac{1}{2}}{3/10} = \frac{1}{6}.$$

(c) Suppose we pick one marble from box 1 and without looking at its color we put it aside. Then we pick another marble from box 1. What is the probability that the second marble is blue?

B_1: first marble is blue A " B_2: first marble is red

$$\mathbb{P}(A) = \mathbb{P}(A \mid B_1)\,\mathbb{P}(B_1) + \mathbb{P}(A \mid B_2)\,\mathbb{P}(B_2)$$

$$= \frac{99}{999}\cdot\frac{1}{10} + \frac{100}{999}\cdot\frac{9}{10}.$$

$$= \frac{1}{10}.$$

"symmetry"

2 Duelling Meteorologists

Tom is a meteorologist in New York. On days when it snows, Tom correctly predicts the snow 70% of the time. When it doesn't snow, he correctly predicts no snow 95% of the time. In New York, it snows on 10% of all days.

(a) If Tom says that it is going to snow, what is the probability it will actually snow?

S: actually snows   T: Tom predicts snow

$$\mathbb{P}(S \mid T) = \frac{\mathbb{P}(S \cap T)}{\mathbb{P}(T)} = \frac{\mathbb{P}(S\cap T)}{\mathbb{P}(S \cap T) + \mathbb{P}(\bar{S} \cap T)} = \frac{0.1 \cdot 0.7}{0.1 \cdot 0.7 + .9 \cdot .05} = \boxed{\frac{14}{23}}$$

(b) Let A be the event that, on a given day, Tom predicts the weather correctly. What is $\mathbb{P}(A)$?

$$\mathbb{P}(A) = \mathbb{P}(S \cap T) + \mathbb{P}(\bar{S} \cap \bar{T}) = \frac{1}{10} \cdot \frac{7}{10} + \frac{9}{10} \cdot \frac{19}{20} = 37/40$$

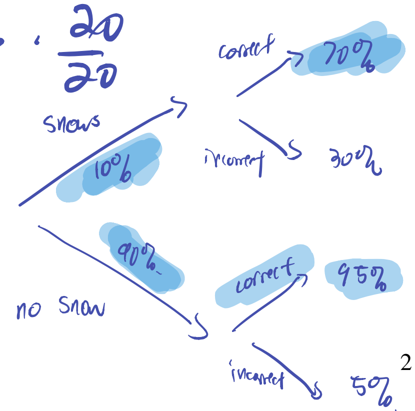
(c) Tom's friend Jerry is a meteorologist in Alaska. Jerry claims that she is a better meteorologist than Tom even though her overall accuracy is lower. After looking at their records, you determine that Jerry is indeed better than Tom at predicting snow on snowy days and sun on sunny day. Give an instance of the situation described above. *Hint: what is the weather like in Alaska?*

→ weighted average: "Simpson's Paradox".

$\mathbb{P}(S) \cdot \mathbb{P}(\text{predict correct} \mid \text{Snow})$

$$\text{Tom's accuracy: } \underbrace{\frac{1}{10}}_{\text{snow}} \cdot \overset{\text{g.c.}}{\frac{7}{10}} + \underbrace{\frac{9}{10}}_{\text{no snow}} \cdot \overset{\text{g.c.}}{\frac{19}{20}} = \frac{37}{40} \approx 92.5\%$$

$$\text{Jerry's accuracy: } \frac{99}{100} \cdot \frac{8}{10} + \frac{1}{100} \cdot \frac{20}{20} = 79\% + 1\% = 80\%$$



80%   100%

70%   95%