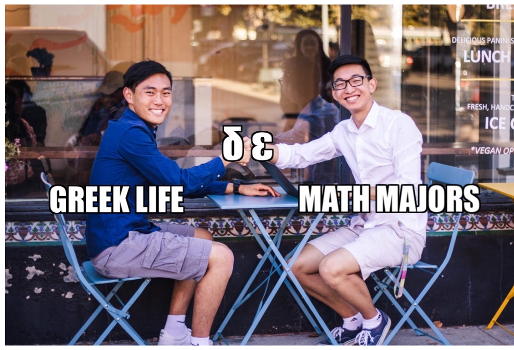
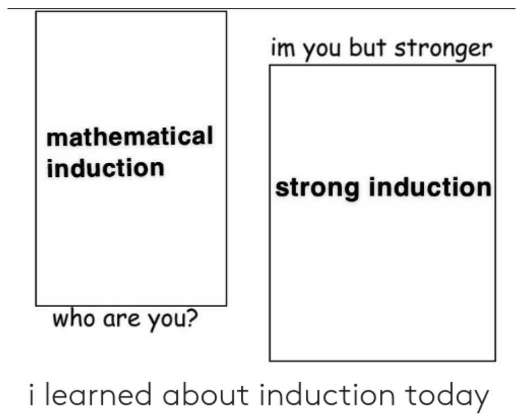
Dis 01B: Induction.

- HW1, Vitamin 1 Due Today!
- bit.ly/tyler-dis1b
- Form study groups, reach out to ppl, etc.
- Happy Labor Day Weekend!





Dis 1A Survey Results

¡ICE CREAM!

"coffee" - literally everyone (the correct answer)

"macapuno (baby coconut)" - noah (i've never heard of this b4?)

"avocado" - khushi (sounds exotic)

"early grey/green tea! :-) but mango + chamoy is p bomb too" - grace (ice cream connoisseur out here y'all)

FUN FACT?

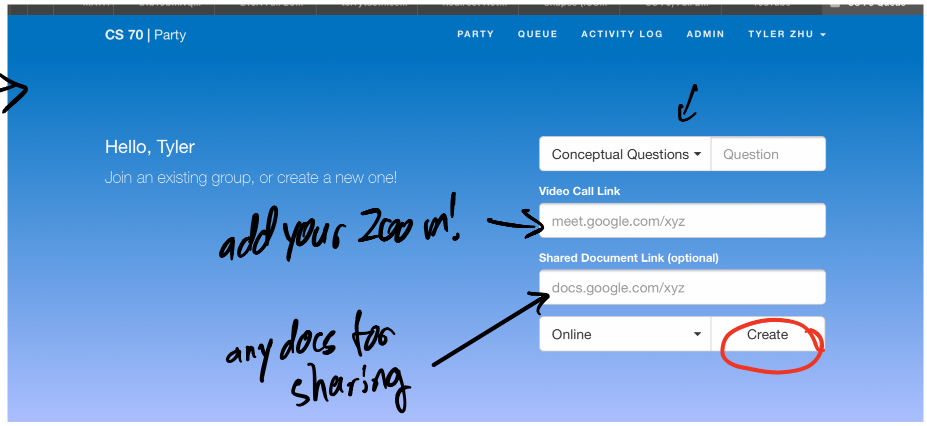
"been on a 100+ mile backpacking trip" - shaylan

"once ate 38 potstickers in 3 minutes for an eating competition" - james

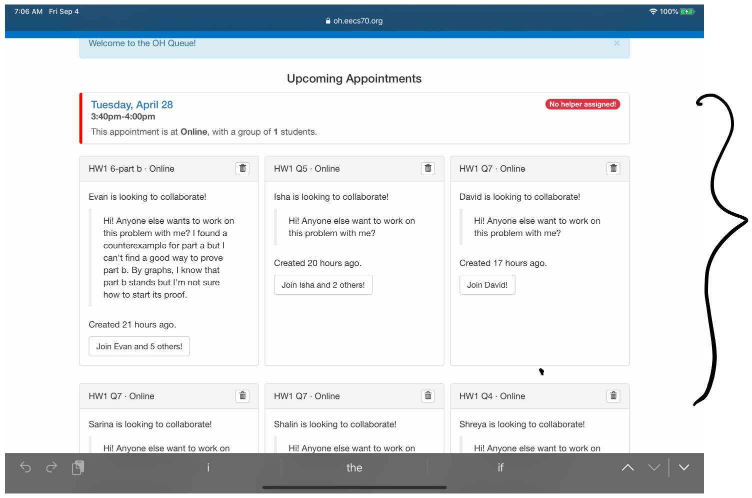
"one of my fav hobbies is crawfishing" - richard

"ran to a drake bell concert at the last minute" - anne oh.eecs70.org

Landing Page



or feel free to join someone else's party w/ the same q.



Review

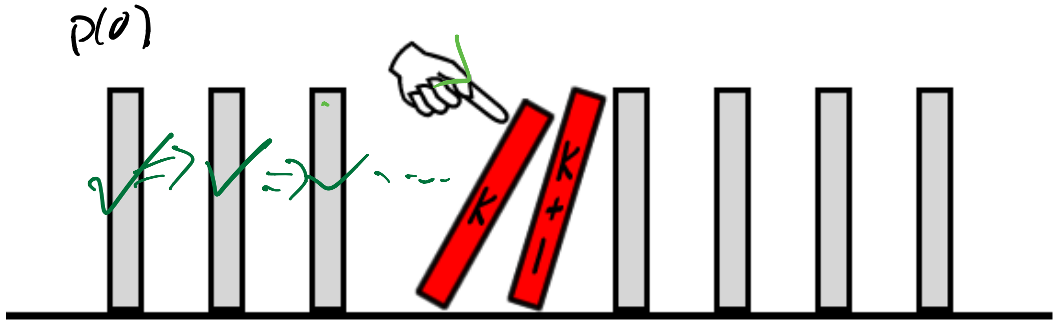
Goal: Prove $(\forall x \in \mathbb{N})(P(x))$ $\quad x = \boxed{0}, 1, 2, \ldots$

Base Case: Show $P(0)$ is true

Induction Hypothesis: Assume that $P(k)$ is true for some $k \geq 0$.

Inductive Step: Prove $P(k+1)$ is true, showing $P(k) \Rightarrow P(k+1)$.

$P(0)$



1 Induction

Prove the following using induction:

(a) Let a and b be integers with $a \neq b$. For all natural numbers $n \geq 1$, $(a^n - b^n)$ is divisible by $(a-b)$.

$a-b \mid a^n - b^n$

Base Case: $n=1$, $a-b \mid a-b$. ✓.

IH: Assume $a-b \mid a^k - b^k$ (for some $k \in \mathbb{N}$), all vals $0,1,\ldots,k$.

IS: WTS $a-b \mid a^{k+1} - b^{k+1}$

$a^k - b^k = (a-b)q$.

$a^{k+1} - b^{k+1} = a \cdot a^k - b \cdot b^k = a \cdot ((a-b)q + b^k)$

$- b(a^k - (a-b) \cdot q)$.

$= a(a-b)q + a \cdot b^k - b \cdot a^k + b \cdot (a-b) \cdot q$

$a \cdot b^k - b \cdot a^k = ab(b^{k-1} - a^{k-1}) = -ab(a^{k-1} - b^{k-1})$

$P(k) \Rightarrow P(k+1)$

$P(k-1), P(k) \Rightarrow P(k+1)$

$$\boxed{a^{k+1} - b^{k+1} = a(a^k - b^k) + b^k(a-b)}$$

(b) For all natural numbers n, $(2n)! \leq 2^{2n}(n!)^2$. [Note that 0! is defined to be 1.]

$6! = 6 \cdot 5 \cdot 4!$

BC: $n=0$. $(2 \cdot 0)! = 1 \leq 2^{2 \cdot 0}(0!)^2 = 1$.

IH: $(2k)! \leq 2^{2k}(k!)^2$.

IS: $(2(k+1))! = 2(k+1) \cdot (2k+1) \cdot (2k)!$

(IH) $\leq 2^{2k}(k!)^2 \cdot 2 \cdot (k+1) \cdot (2k+1)$

$= 2^{2k+1} \cdot (k+1)! \cdot k! \cdot (2k+1)$.

$\leq 2^{2k+1} \cdot (k+1)! \cdot k! \cdot (2k+2)$

$\leq 2^{2k+2} \cdot (k+1)! \cdot (k+1)!$
$= 2^{2(k+1)} \cdot (k+1)!^2$

3 Binary Numbers

Prove that every positive integer n can be written in binary. In other words, prove that we can write

$$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \cdots + c_1 \cdot 2^1 + c_0 \cdot 2^0,$$

where $k \in \mathbb{N}$ and $c_k \in \{0,1\}$.

$10 = 1010 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2 + 0 \cdot 1$
$= 8 + 2 = 10.$

BC: $n=1$ $n = 1 = 1 \cdot 2^0 = 1$ $c_0 = 1,\ c_i = 0 \ldots$ ✓

IH: Assume true for n, i.e. $\exists\ c_k \in \{0,1\}$
$n = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} \ldots + c_1 \cdot 2^1 + c_0 \cdot 2^0.$

IS. WTS $n+1$ has similar form.

$n+1 = c_k \cdot 2^k + c_{k-1} \cdot 2^{k-1} + \cdots + c_1 \cdot 2^1 + (c_0 + 1) \cdot 2^0.$

Case 1 $c_0 = 0$: ↑ all good. $\boxed{n \to n+1}$

Case 2 $c_0 = 1$. WTS $n+1$ has the form, $n \to n+1$ doesn't work.

$10 = 101\cancel{0}_2$. $\frac{n+1}{2} = c_k' \cdot 2^k + c_{k-1}' \cdot 2^{k-1} + \ldots + c_1' \cdot 2^1 + c_0' \cdot 2^0.$

$5 = 101_2$. $n+1 = c_k' \cdot 2^{k+1} + c_{k-1}' \cdot 2^k + \ldots + c_1' \cdot 2^2 + c_0' \cdot 2^1$
$+ 0 \cdot 2^0.$

$\boxed{\frac{n+1}{2} \to n+1}$