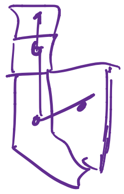
Dis 03A: Graphs

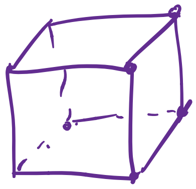
1 Cube Dual

4-color theorem

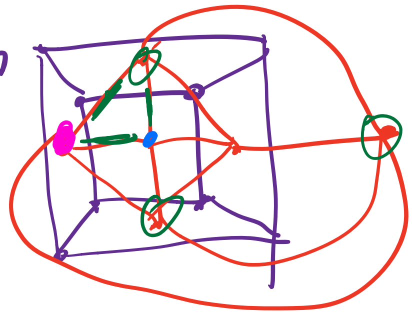


We define a graph G by letting the vertices be the corners of a cube and having edges connecting adjacent corners. Define the *dual* of a planar graph G to be a graph G', constructed by replacing each face in G with a vertex, and an edge between every vertex in G' if the respective faces are adjacent in G.

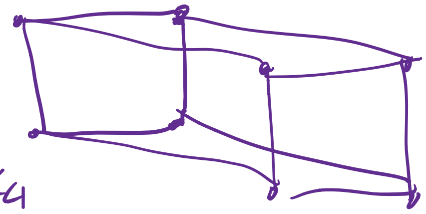
(a) Draw a planar representation of G and the corresponding dual graph. Is the dual graph planar? (Hint: think about the act of drawing the dual)



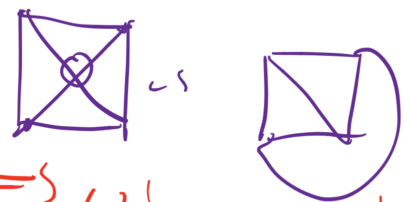
G



G'



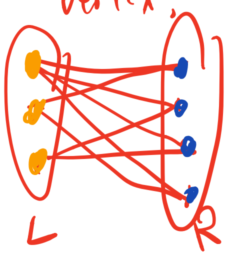
K4



(b) Is G' bipartite?

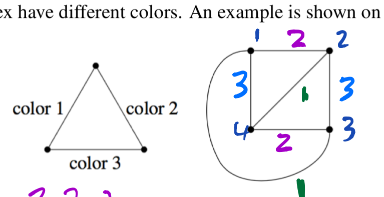
G' bipartite ⟺ G' is 2 vertex colorable ⟺ G''s vertices can be partitioned into two sets (L & R) so that all edges in G' go b/w the sets (L & R have no edges in b/w)

⟺ G' has no odd cycles.



3 Edge Colorings

An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.



(a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers 1, 2, 3 for colors. A figure is shown on the right.)

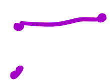
(b) Prove that any graph with maximum degree $d \geq 1$ can be edge colored with $2d-1$ colors.

(induction).

(c) Show that a tree can be edge colored with d colors where d is the maximum degree of any vertex.

induct on # of edges (not max degree)

Base: n=1,

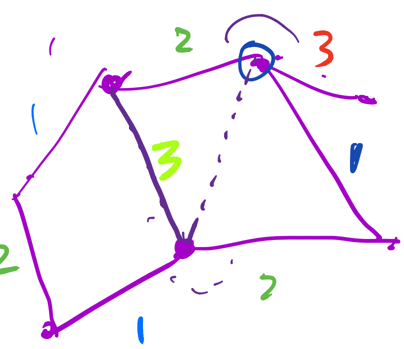


Δ=1 ✓ 1 color

IH: Assume true n=k edges, max deg d

IS: G w/ k+1 edges, max deg d.

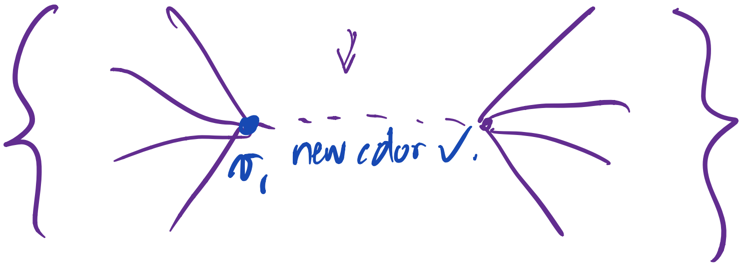
8 edges.



→ k edges ⇒ 2d'-1

d' = new max degree

$a \leq d-1$ vertices



$\leq d-1$ vertices

$\underline{d-1} + d-1 = 2d-2.$ } max # color used to color these edges.

(c) Show that a tree can be edge colored with d colors where d is the maximum degree of any vertex.

2 True or False

(a) Any pair of vertices in a tree are connected by exactly one path.

(b) Adding an edge between two vertices of a tree creates a new cycle.

(c) Adding an edge in a connected graph creates exactly one new cycle.