# Dis 02B: Graphs

- HW2, Vitamin 2 due tonight
- Dis Swap Form due tonight
- No-HW option form due Sunday

## Six degrees of separation

From Wikipedia, the free encyclopedia

*For the 2012 song by The Script, see Six Degrees of Separation (song).*

*"Six degrees" redirects here. It is not to be confused with Six degrees of freedom. For other uses, see Six*

**Six degrees of separation** is the idea that all people are six, or fewer, social connections away from each other. Also known as the 6 Handshakes rule. As a result, a chain of "a friend of a friend" statements can be made to connect any two people in a maximum of six steps. It was originally set out by Frigyes Karinthy in 1929 and popularized in an eponymous 1990 play written by John Guare. It is sometimes generalized to the average social distance being logarithmic in the size of the population.

## Review.

A walk is .. a traversal, vertices & edges can repeat

A path is .. a walk w/ no repeated vertices/edges.

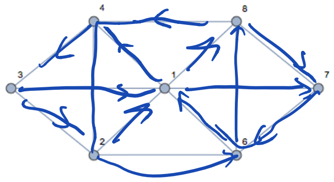
A tour is .. a walk where start & end are same and no repeated edges.

A cycle is ... a path where start & end are same.

| | non-"cycle" | "cycle" |
|---|---|---|
| repeat | walk | tour |
| non-repeats | path | cycle |

An Eulerian walk/tour is .. one where reach every edge once,

## 1 Eulerian Tour and Eulerian Walk




(a) Is there an Eulerian tour in the graph above? If no, give justification. If yes, provide an example.

(b) Is there an Eulerian walk in the graph above? An Eulerian walk is a walk that uses each edge exactly once. If no, give justification. If yes, provide an example.

(c) What is the condition that there is an Eulerian walk in an undirected graph? Briefly justify your answer.

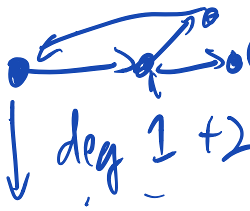
| | non-"cycle" | "cycle" |
|---|---|---|
| repeat | walk | tour |
| non-repeats | path | cycle |

(a). No, every nodeneeds even degree.

(b). 3 - 1 - 4 - 3 - 2 - 1 - 8 - 4 - 2 - 6 - 8 - 7 - 6 - 1 - 7.

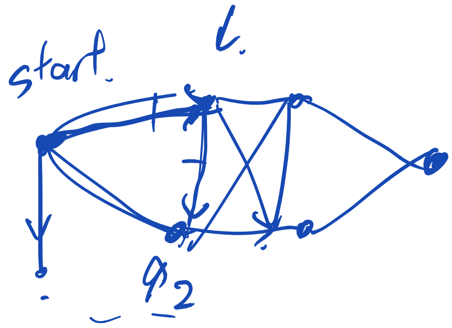
(c). 0 or 2 odd deg nodes.

start



end → deg 1.

deg 1 +2 +2



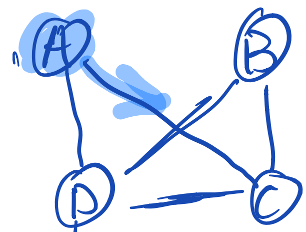
## 2 Banquet Arrangement

$n = 4$ $m = 2$.

In the words of the great Ana Lynch, "Let's have a kiki."

Suppose $n$ people are attending a kiki, and each of them has at least $m$ friends ($2 \le m \le n$), where friendship is mutual. Prove that we can put at least $m+1$ of the attendants on the same round table, so that each person sits next to his or her friends on both sides.

Alice (A, C)
Bob (A, D)
Charles (B, C)
David. (B, D).




Let G have n nodes, and each node has deg m. Prove $\exists$ a cycle of length at least m+1.

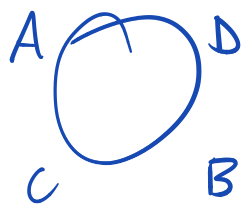
1. every node connected to m other nodes

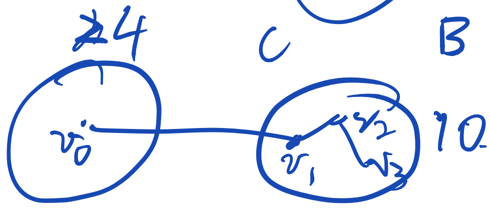
$\ell \ge m$

go back to most recent neighbor of $v_0$. H $(v_\ell, v_0)$, done.

$P = v_0 v_1 v_2 \cdots v_\ell$

m. m-1 m-2 no more neighbors




$P' = v_\ell v_{\ell-1} \cdots v_k$, ($v_k$ earliest neighbor $v$)

$m = 3$.

$P = v_0 v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9$

1

$P' = v_9 v_8 v_7 \cdots v_4 v_3 v_2$.

## 3 Not everything is normal: Odd-Degree Vertices

**Claim:** Let $G = (V, E)$ be an undirected graph. The number of vertices of $G$ that have odd degree is even.

Prove the claim above using:

(i) Direct proof (e.g., counting the number of edges in $G$). *Hint: in lecture, we proved that* $\sum_{v \in V} \deg v = 2|E|$.

(ii) Induction on $m = |E|$ (number of edges)

(iii) Induction on $n = |V|$ (number of vertices)