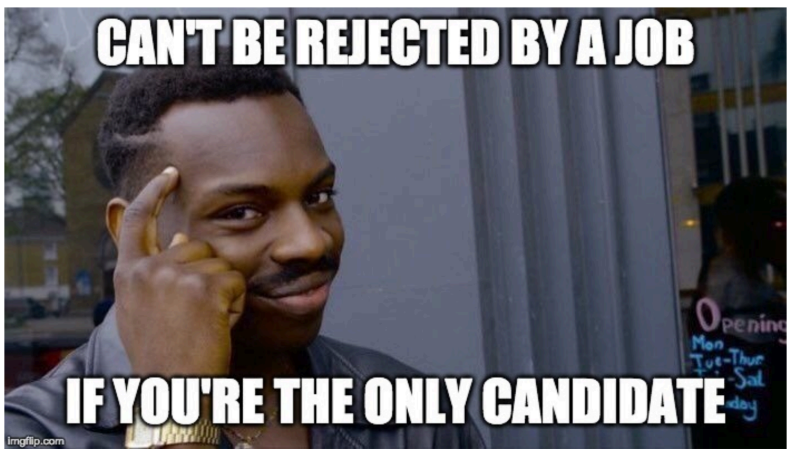
Dis 2A: Stable Matching

- fill out dis swap form
- MT1 is on Tuesday, 10/13 from 8-10 PM PST
-

- check out our spotify playlist!
  - should we make a specific vibe?
- cs 70 social things?

we're always trying to improve you know (improvement lemma)

if candidates are proposing... (improvement lemma)



review

terminology

- An **instance** of a stable matching is a set of preference lists of jobs and candidates.
- A **matching** for a stable matching is a set of candidate-job couples (C_i, J_i).
- A **rogue couple** is a pair (C, J) that prefer each other over their current partners.
- A **stable** matching is a matching without any rogue couples.
- A **(job/candidate) optimal** matching is one where the (jobs/candidates) receive their highest preferences of *all* stable matchings.

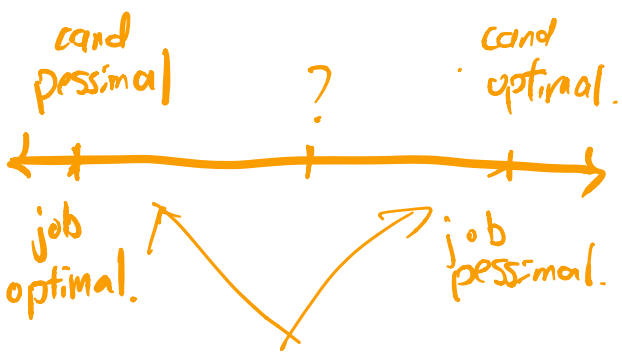
example (n=3)

C	J		
1	A	B	C
2	B	A	C
3	A	B	C

J	C		
A	2	1	3
B	1	2	3
C	1	2	3

(A,1), (B,2), (C,3).

~ the spectrum ~



the propose-and-reject algorithm

Recall that the propose-and-reject algorithm performs three stages every day until termination:

(a) **Morning**: Every job proposes to the best candidate who has yet to reject the job yet.

(b) **Afternoon**: Each candidate rejects all jobs proposed to her except for her favorite job, which she keeps on a string.

(c) **Evening**: Each job crosses off the candidate that rejected them, if any.

- *Candidate Improvement Lemma*: The job a candidate has on her string can only get better.

1 Stable Matching

Consider the set of candidates $C = \{1, 2, 3\}$ and the set of jobs $J = \{A, B, C\}$ with the following preferences.

C	J		
1	A	B	C
2	B	A	C
3	A	B	C

J	C		
A	2	1	3
B	1	2	3
C	1	2	3

Run the applicant propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work)

reminder:

Recall that the propose-and-reject algorithm performs three stages every day until termination:

(a) **Morning**: Every job proposes to the best candidate who has yet to reject the job yet.

(b) **Afternoon**: Each candidate rejects all jobs proposed to her except for her favorite job, which she keeps on a string.

(c) **Evening**: Each job crosses off the candidate that rejected them, if any.

- *Candidate Improvement Lemma*: The job a candidate has on her string can only get better.

Jobs	Day 1	Day 2	Day 3
A	1, 3	1	1
B	2	2, 3	2
C			3

(A,1), (B,2), (C,3)

cand opt

stable

follow up: how many matchings are there in total?

C	D1	D2	D3
1	B, C	B	B
2	A	A, C	A
3			C

(B,1), (A,2), (C,3)

job opt.

2 Good, Better, Best

In a particular instance of the stable ~~marriage~~ matching problem with n applicants and n jobs, it turns out that there are exactly three distinct stable matchings, S_1, S_2, and S_3. Also, each applicant m has a different partner in the three matchings. Therefore each applicant has a clear preference ordering of the three matchings (according to the ranking of his partners in his preference list). Now, suppose for applicant m_1, this order is $S_1 > S_2 > S_3$.

Prove that every applicant has the same preference ordering $S_1 > S_2 > S_3$.

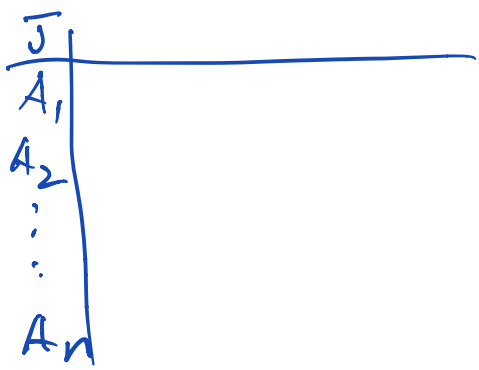
Hint: Recall that a applicant-optimal matching always exists and can be generated using applicant proposes matching algorithm. By reversing the roles of stable matching algorithm, what other matching can we generate?

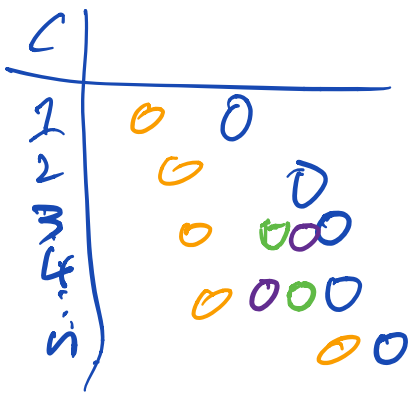
in this instance, have >2 matchings, ∃

applicant opt ? applicant pessimal => every applicant has the same pref order.

∀c m_1 has $S_1 > S_2 > S_3$, everyone has $S_1 > S_2 > S_3$. ∎

follow up: 4 matchings?





S_2

S_3